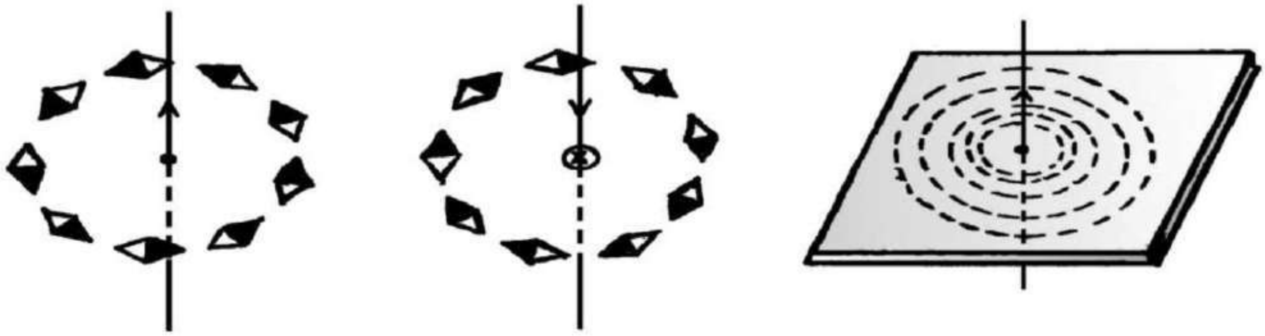
Unit-4
Moving Charges & Magnetism

In the year 1820 Oersted realised that electricity and magnetism were intimately related. He shows that the electric current through a straight wire causes noticeable deflection of the magnetic compass needle held near the wire.

He also found that the magnetic needle is aligned tangentially to an imaginary circle which has the straight current carrying wire as its centre and has its plane perpendicular to the wire.

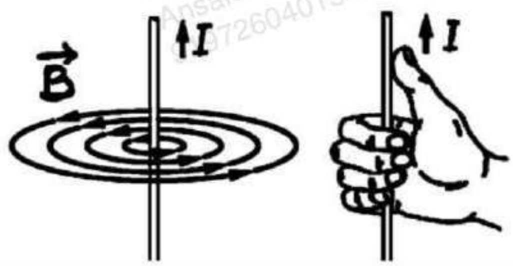


If the direction of the current in wire is reversed, the direction of deflection of the magnetic needle is also reversed. The deflection increases on increasing the current in the wire or bringing the magnetic compass needle closer to the wire.

Magnetic Field

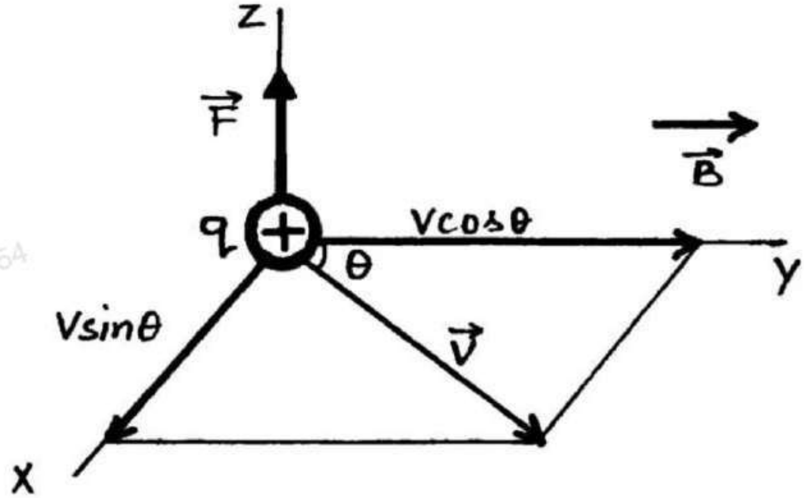
The magnetic field is a space around a conductor carrying current or the space around a magnet in which its magnetic effect can be felt.

It is a vector quantity and is denoted by $\vec{B}$.



Force on a moving charge in a magnetic field

Consider a positive charge q moving in a uniform magnetic field $\vec{B}$, with a velocity $\vec{V}$. Let the angle between $\vec{V}$ and $\vec{B}$ be θ



Due to interaction between the magnetic field produced due to moving charge (i.e current) and magnetic field applied, the charge q then experiences a force, which depends on the following factors -

$$F \propto q$$

$$F \propto V \sin\theta$$

$$F \propto B$$

so by combining the above factor, we get

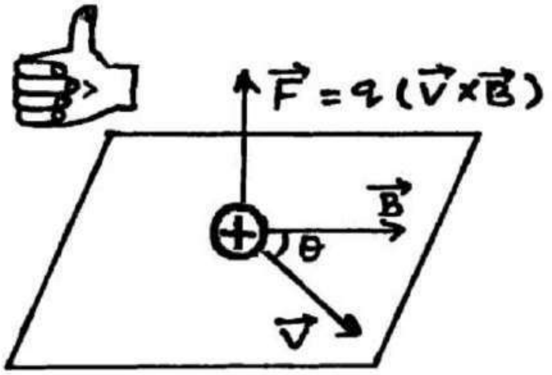
$$F = KqVB\sin\theta$$

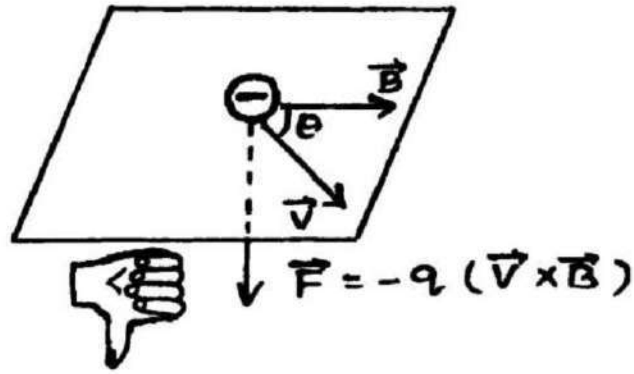
Here K is a constant. Its value is found to be 1.

$$\boxed{\begin{aligned} F &= qVB\sin\theta \\ \vec{F} &= q(\vec{V} \times \vec{B}) \end{aligned}}$$

The direction of $\vec{F}$ is the direction of cross-product of velocity $\vec{V}$ and magnetic field $\vec{B}$, which is perpendicular to the plane containing $\vec{V}$ and $\vec{B}$.

Its direction is given by the 'Right hand Rule'.





Unit - SI unit of $\vec{B}$ is Tesla (T) or Weber / metre^2 or $N A^{-1} m^{-1}$.

Dimensions of $\vec{B}$ = $[M^1 A^{-1} T^{-2}]$

1 Tesla - The magnetic field $\vec{B}$ at a point is said to be 1 tesla if a charge of one Coulomb while moving at right angle to a magnetic field with a velocity of 1 m/s experience a force of 1 Newton at that point.

$$B = \frac{F}{qv\sin\theta} \quad \text{so} \quad 1T = \frac{1N}{1C \times 1 m/s} = 1 N A^{-1} m^{-1}$$

Q. An α particle of mass 6.65×10^{-27} Kg is travelling at right angles to a magnetic field with a speed of 6×10^{5} m/s. The strength of the magnetic field is 0.2 T. Calculate the force on the α particle and its acceleration.

Sol. Given that- $m = 6.65 \times 10^{-27}$ Kg

$q = 2e = 2 \times 1.6 \times 10^{-19}$ C, $V = 6 \times 10^{5}$ m/s

$B = 0.2$ T and $\theta = 90^\circ$

So the force on the α particle in magnetic field is

$$F = qVB\sin\theta$$

$$F = 2 \times 1.6 \times 10^{-19} \times 6 \times 10^{5} \times 0.2 \times \sin 90^\circ$$

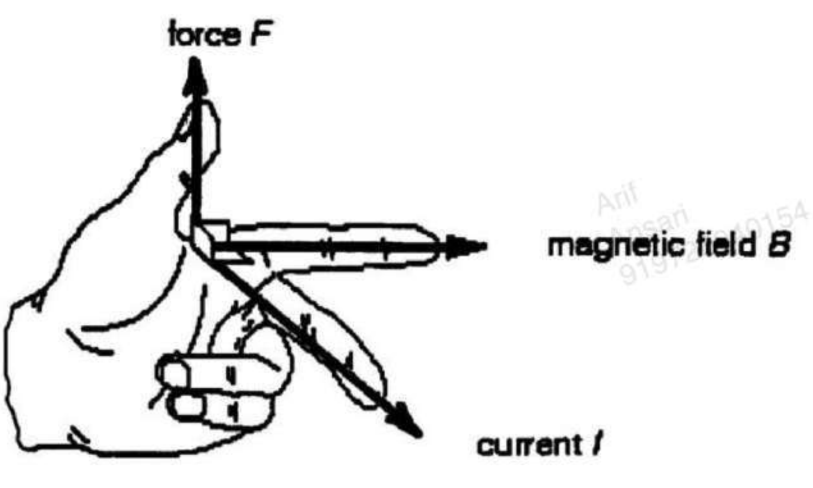
$$F = 3.84 \times 10^{-14}\ N$$

and acceleration of the α particle is-

$$a = \frac{F}{m} = \frac{3.84 \times 10^{-14}}{6.65 \times 10^{-27}} = 5.77 \times 10^{12}\ m/s^2$$

Fleming Left hand Rule

If we stretch the first finger, the central finger and the thumb of left hand mutually perpendicular to each other such that the first finger points towards the direction of magnetic field, the central finger points towards the direction of electric current (motion of the positive charge) then the thumb represents the direction of the force experienced by the charged particle.



Q. All India 2011 Write the expression for Lorentz magnetic force on a particle of charge q moving with velocity V in a magnetic field B. Show that no work is done by this force on the charged particle.

Sol. Lorentz force always acts along the direction perpendicular to the direction of velocity of the particle.

Magnetic Lorentz force $F_m = q(V \times B)$

$\because \quad F \perp V$

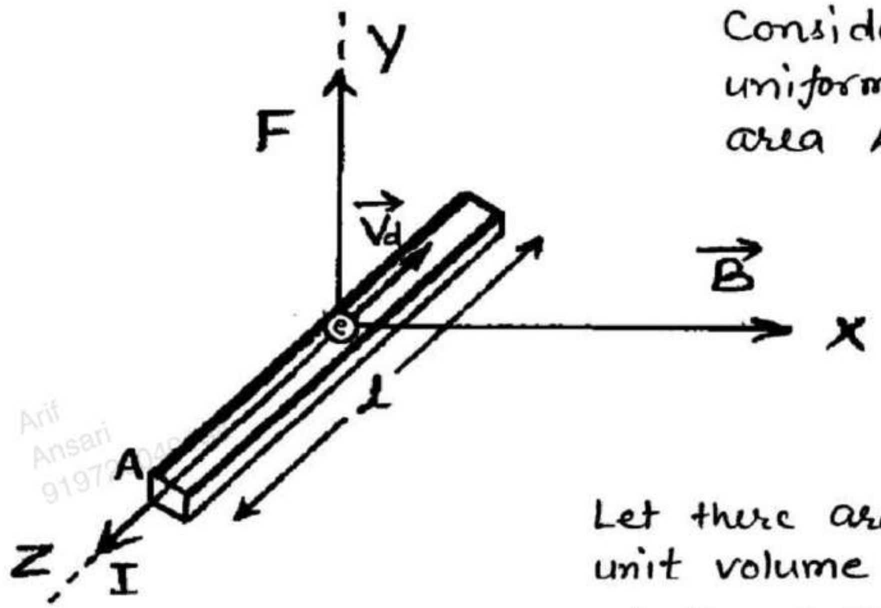
Force is perpendicular to displacement made by charge particle.

$\therefore \quad W = F d \cos 90^\circ = 0$ [Force & displacement are perpendicular

$W = 0$

So no work is done by magnetic Lorentz force on charge particle.

Magnetic Force on a Current Carrying Conductor



Consider a wire of uniform cross-sectional area A and length l.

Let there are n electrons per unit volume and the drift velocity of the electrons is V_d.

Total number of electrons in it is $= nAl$

So the force on these electrons is -

$$F = -(nAle)\, V_d B \sin\theta \qquad [q = -nAle]$$

$$F = (-neAV_d)\, lB \sin\theta \qquad [I = -nAeV_d]$$

$$\boxed{\begin{array}{l} F = ILB\sin\theta \\ \vec{F} = I(\vec{l} \times \vec{B}) \end{array}}$$

or

Here $\vec{l}$ is a vector of magnitude l and with direction towards the current I.

★ Here the direction of the $\vec{F}$ is perpendicular to the plane containing $\vec{B}$ and $\vec{l}$.

(i) If $\theta = 0°$ or $180°$ then $\vec{F} = 0$

(ii) If $\theta = 90°$ then $\vec{F}_m = IBL$

★ The direction of Force can be given by the "Fleming's Left hand Rule".

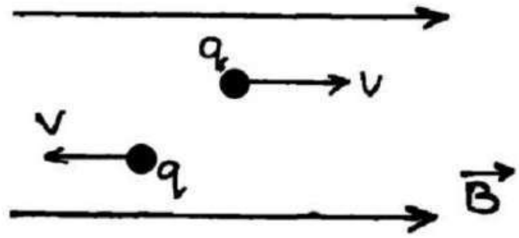
Motion in a Magnetic Field

When a moving charge particle moves in parallel or antiparallel direction to the magnetic field then it does not feel any force.

Case 1- When the charged particle is moving along or in opposite to the magnetic field ($\theta = 0°$ or $180°$)

$$F = qVB \sin 0°$$

$$F = 0$$



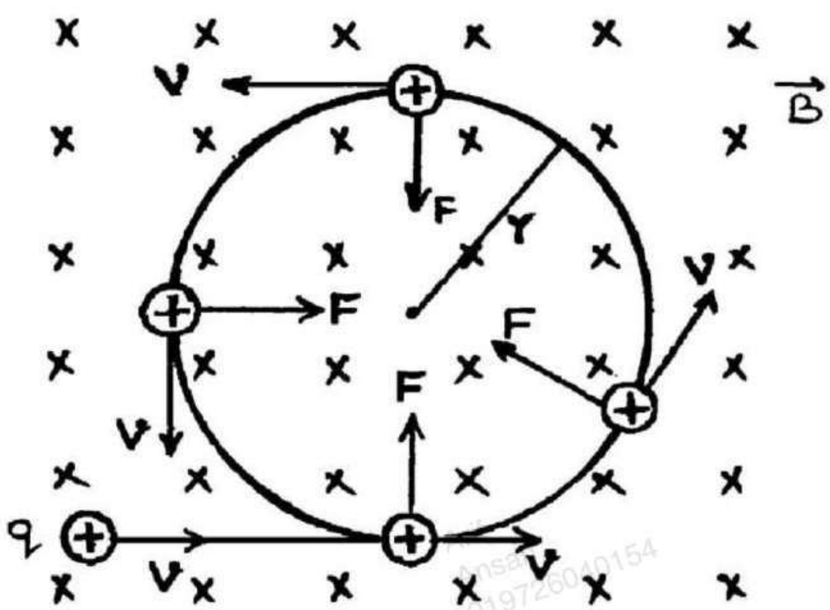
Path of the particle → Linear (straight line)

Case 2- When charge particle moves at right angle to the magnetic field

$$\vec{F} = q(\vec{V} \times \vec{B})$$

$$= qVB \sin 90°$$

$$F = qVB$$



This force is always perpendicular to the direction of motion and the magnetic field B so the charge will describe a circular path.

Here the force F provides the required centripetal force necessary for motion along a circular path of radius r.

$$qvB = \frac{mv^2}{r}$$

so $\quad r = \frac{mv^2}{qvB} \quad \Rightarrow \quad \boxed{r = \frac{mv}{qB}}$

The time period of revolution of the particle is

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v} \times \frac{mv}{qB}$$

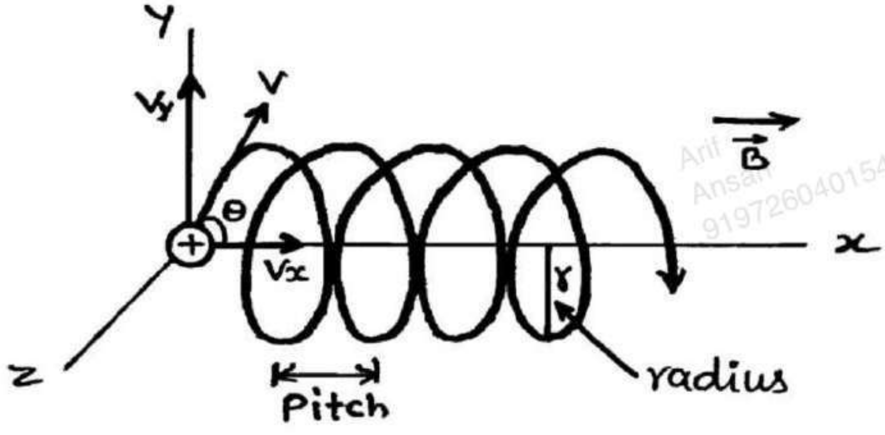
so $\quad \boxed{T = \frac{2\pi m}{qB}}$

The angular velocity of rotation of the particle

$\omega = 2\pi\nu = \frac{2\pi}{T} \quad \Rightarrow \quad \boxed{\omega = \frac{qB}{m}}$

Here we can see that T and ω does not depend upon velocity $\vec{v}$ of the particle. It means all the charged particles having the same specific charge (q/m) but moving with different velocities at a point, will complete their circular paths in the same time,

<u>Case 3</u> - When a charge particle at any angle (other than 0°, 90° and 180°) to the magnetic field



Now $V_x = V\cos\theta$ is the factor of $\vec{V}$ along the direction of $\vec{B}$ and $V_y = V\sin\theta$ is the factor of $\vec{V}$ in perpendicular direction to $\vec{B}$.

For component of velocity $V\cos\theta$, there will be no force on the charged particle in the magnetic field, because the angle between $\vec{V_1}$ and $\vec{B}$ is zero. Thus the charged particle covers the linear distance in direction of the magnetic field with a constant speed $V\cos\theta$.

Also the component of velocity $V\sin\theta$ is in the perpendicular direction to the magnetic field $\vec{B}$.
Due to it, the charged particle move on a circular path in the magnetic field.

The required centripetal force for motion along a circular path of radius r is provided by the magnetic force as:-

$$\frac{m(V\sin\theta)^2}{r} = q(V\sin\theta)B\sin 90^\circ$$

$$\boxed{r = \frac{mV\sin\theta}{qB}}$$

and the time period of the particle is -

$$\boxed{T = \frac{2\pi r}{V\sin\theta} = \frac{2\pi m}{qB}}$$

Therefore, under the combined effect of the two component velocities, the charged particle in magnetic field will cover linear path as well as circular path hence the path of the charged particle will be helical whose axis is parallel to the direction of magnetic field.

Pitch - The linear distance covered by the charged particle in the magnetic field in one revolution of its circular path is known as the pitch of helix.

$$\text{Pitch} \quad P = V\cos\theta \times T$$

$$\boxed{P = \frac{2\pi m v \cos\theta}{qB}}$$

Q. (CBSE 2010) A deutron and a proton moving with the same speed enter the same magnetic field region at right angles to the direction of the field. Show the trajectories followed by the two particles in the magnetic field. Find the ratio of the radius of the circular paths which the two particles may describe.

Sol. When a charged particle enters in the magnetic field at right angle, then the particle follows a circular path.

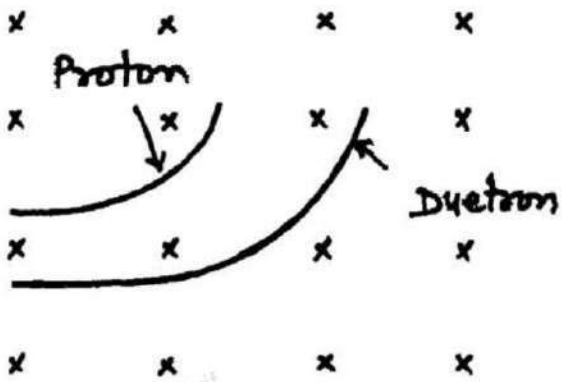
Radius of the circular path $\quad r = \frac{mv}{qB}$

$$\because \quad \frac{r_p}{r_d} = \frac{m_p}{m_d}$$

$$\because \quad m_d = 2m_p$$

$$\text{so} \quad r_d = 2r_p$$

$$\text{or} \quad r_d : r_p = 2 : 1$$



Note - Smaller the radius, greater the curvature and vice-versa. This is why the protons path has got greater curvature.

Q. An electron of mass 0.9×10^{-30} Kg under the action of magnetic field moves in a circle of radius 2 cm at a speed of 3×10^6 m/s. If a proton of mass 1.8×10^{-27} Kg, were to move in a circle of the same radius in the same magnetic field, then find its speed.

Sol: We Know that $qvB = \frac{mv^2}{r}$

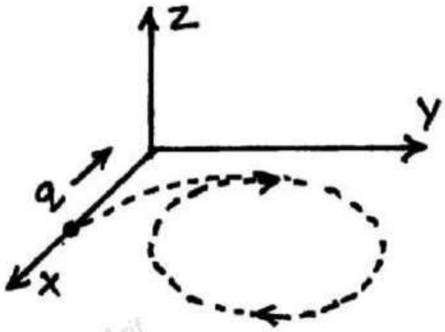
or $v = \frac{qrB}{m}$

So $\frac{V_p}{V_e} = \frac{q_p r_p B/m_p}{q_e r_e B/m_e} = \frac{m_e}{m_p}$

$$V_p = V_e \times \frac{m_e}{m_p} = \frac{3 \times 10^6 \times 9 \times 10^{-31}}{1.8 \times 10^{-27}} = 1.5 \times 10^3 \text{ m/s}.$$

Q. (CBSE 2009) A charge q moving along the x-axis with a velocity v is subjected to a uniform magnetic field B acting along the z-axis
(a) Trace Trajectory
(b) Does, the charge particle gain kinetic energy as it enters the magnetic field?

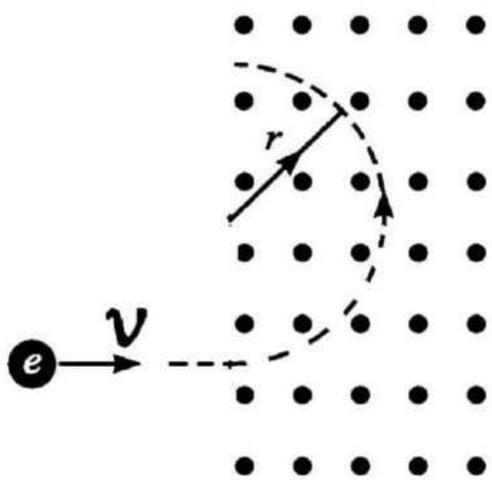
Sol: (a) As the charged particle is moving perpendicular to the magnetic field, so it will perform circular motion in x-y plane.



(b) No, the charged particle does not gain any Kinetic Energy because the Lorentz force acting on it does not perform any work as $F_m \perp v$.

Q. CBSE 2015 An electron moving horizontally with a velocity of 4×10^4 m/s enters a region of uniform magnetic field of 10^{-5} T acting vertically upwards as shown in figure. Draw its trajectory and find the time it takes to come out of the magnetic field.

Sol. From fleming's left hand Rule the electron deflect in anticlockwise direction and describe a semicircular path.



Here magnetic force provides a centripetal force, so

$$evB = \frac{mv^2}{r}$$

$$v = \frac{eBr}{m}$$

Time taken $T = \frac{\pi r}{v}$

$$T = \frac{\pi r}{eBr/m} = \frac{\pi m}{eB}$$

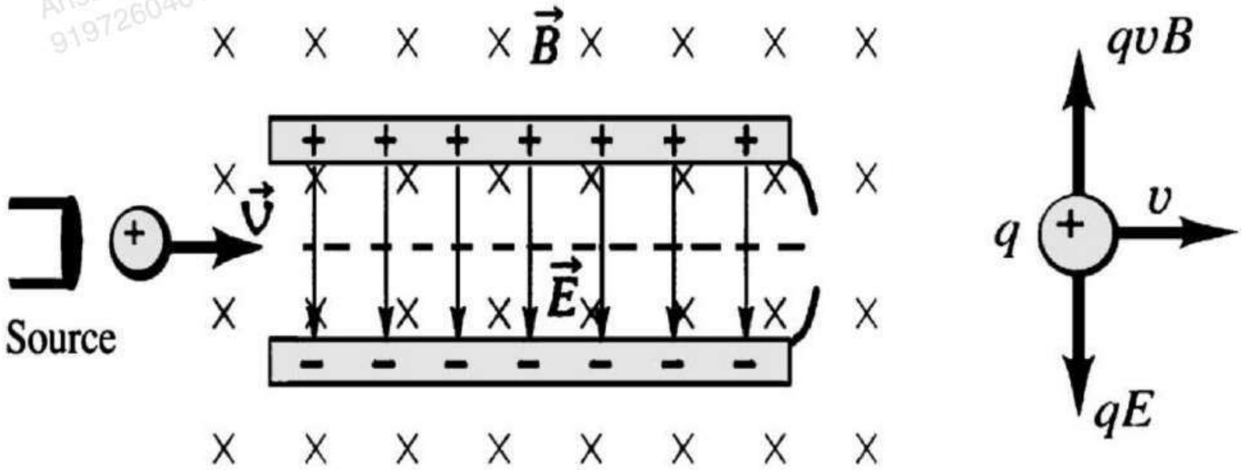
$$T = \frac{3.14 \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19} \times 10^{-5}} = 1.78 \times 10^{-6} \text{ sec.}$$

Velocity Selector

A charge q is moving with velocity $\vec{v}$ in presence of both electric and magnetic fields feels a force given as-

$$\boxed{\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})}$$

Now consider a simple case when electric and magnetic fields are perpendicular to each other and also perpendicular to the velocity of the particle.



Suppose we adjust the value of $\vec{E}$ and $\vec{B}$ such that the magnitudes of the two forces are equal, then the total force on the charge is zero and the charge will move in the fields undeflected.

This happens when-

$$F_E = F_B$$

$$qE = qvB \qquad \text{or} \qquad \boxed{v = \frac{E}{B}}$$

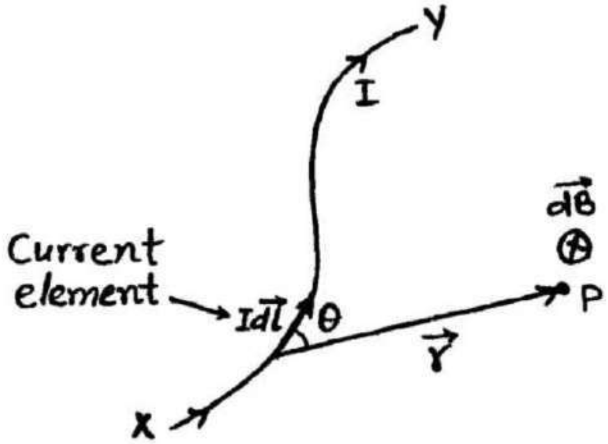
This condition can be used to select charged particles of a particular velocity out of a beam containing charges moving with different speeds. The crossed E and B fields, therefore serve as a velocity selector.

Only the particles with speed E/B pass undeflected through the region of crossed fields.

BIOT SAVART LAW

Let $\vec{r}$ be the position vector of the point P from the current element $I\vec{dl}$ and θ be the angle between $\vec{dl}$ and $\vec{r}$.

The current element $I\vec{dl}$ is a vector. Its direction is tangent to the element and is acting in the direction of current flow in the conductor.



(i) $dB \propto I$ (ii) $dB \propto dl$

(iii) $dB \propto \sin\theta$ (iv) $dB \propto 1/r^2$

Combining these factors, we get -

$$dB \propto \frac{I\,dl\sin\theta}{r^2} \quad \text{or} \quad dB = \frac{K\,I\,dl\sin\theta}{r^2}$$

$$K = \frac{\mu_0}{4\pi} = 10^{-7} \rightarrow \text{In SI Units}$$

So that $$\boxed{dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}}$$

In vector form we may write -

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi}\frac{I(\vec{dl}\times\hat{r})}{r^2}}$$

Here direction of $d\vec{B}$ is represented by 'Right hand Rule'.

Similarities and Dis-similarities between the Biot-Savart's Law & Coulomb's Law -

Similarities -

(i) Both the laws for fields are long range, since in both the laws, the field at a point varies inversely as the square of the distance from the source.

(ii) Both the fields obey superposition principle.

Dis-similarities -

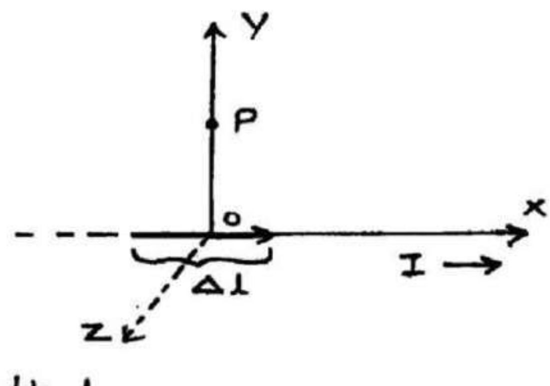
(i) The electrostatic field is acting along the displacement vector while the magnetic field is acting perpendicular to the plane containing the current element Idl and displacement vector $\vec{r}$.

(ii) The Coulomb's Law is independent of angle whereas the Biot-Savart's Law is angle dependent.

(iii) The electrostatic field is produced by a scalar source (q) while magnetic field is produced by a vector source ($Id\vec{l}$)

Q. An element $\Delta l = \Delta x \hat{i}$ is placed at the origin and carries a current $I = 2A$. Find the magnetic field at a point P on the Y axis at a distance of 1 m due to the element $\Delta x = w$ cm. Give also the direction of the field produced.

CBSE 2009



Sol. Biot-Savart Law states that -

$$dB = \frac{\mu_0}{4\pi} \frac{Idl \times \hat{r}}{r^2}$$

Here $Idl = 2 \times w\hat{i}$

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{(2w\hat{i} \times \hat{j})}{(1)^2}$$

Here ($\vec{r} = \hat{j}$)

so ($|\vec{r}| = 1$)

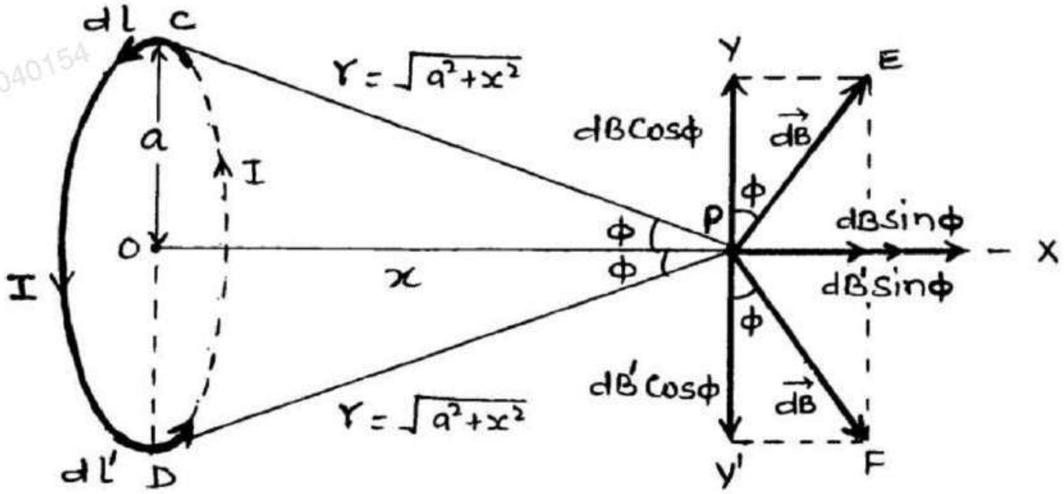
$$dB = \frac{\mu_0 w \hat{k}}{2\pi}$$

or $|dB| = \frac{\mu_0 w}{2\pi}$ (along +z axis)

Magnetic Field on the axis of a Circular Current Loop -

Consider a circular coil of radius a with centre O. Carrying current I. Suppose P is any point on the axis of the circular coil at a distance x from its centre O. ($OP = x$)

Now consider two small element of the coil each of length dl, at C and D which are situated at diametrically opposite edges.



According to the Biot - Savart's Law, the magnetic field at P due to current element $I\vec{dl}$ at C is given by -

$$dB = \frac{\mu_0}{4\pi} \frac{I\,dl \sin 90^\circ}{r^2}$$

$$dB = \frac{\mu_0}{4\pi} \frac{I\,dl}{(a^2+x^2)} \qquad \text{—— (1)}$$

The direction of $d\vec{B}$ is in the plane perpendicular to $\vec{dl}$ and $\vec{r}$ and is directed as given by right hand screw rule i.e. acting along PE $\perp$ to $\vec{CP}$.

Similarly, the magnitude of magnetic field at P due to current element of length dl at D is given by -

$$dB' = \frac{\mu_0}{4\pi} \frac{I\,dl \sin 90^\circ}{r^2}$$

$$dB' = \frac{\mu_0}{4\pi} \frac{I\,dl}{(a^2+x^2)} \qquad \text{—— (2)}$$

Its direction is along $PF \perp DP$.

from eq. ① & ② $dB = dB' = \frac{\mu_0}{4\pi} \frac{I\,dL}{(a^2+x^2)}$

Since the components of the magnetic fields along PY and PY' are equal and opposite, they cancel each other and the components acting along PX are in same direction so they added up.

So the total magnetic field at P due to current through whole circular coil is given by -

$$B = \int dB \sin\phi = \int \frac{\mu_0}{4\pi} \frac{I\,dl \sin\phi}{(a^2+x^2)}$$

$$B = \frac{\mu_0 I}{4\pi(a^2+x^2)} \sin\phi \int dL$$

$$B = \frac{\mu_0 I}{4\pi(a^2+x^2)} \times \frac{a}{\sqrt{a^2+x^2}} \times 2\pi a$$

$\sin\phi = \frac{a}{\sqrt{a^2+x^2}}$

$\int dL = 2\pi a$

$$\boxed{B = \frac{\mu_0}{4\pi} \frac{2\pi a^2 I}{(a^2+x^2)^{3/2}} \quad \text{(along PX)}}$$

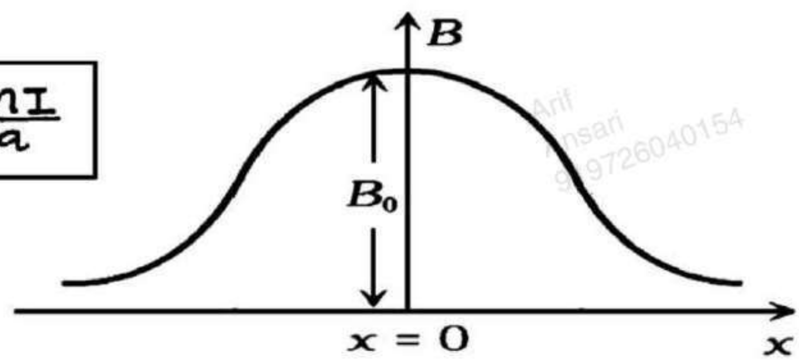
If there are n turns in the coil, then -

$$\boxed{B = \frac{\mu_0}{4\pi} \frac{2\pi n I a^2}{(a^2+x^2)^{3/2}}}$$

Special Case -

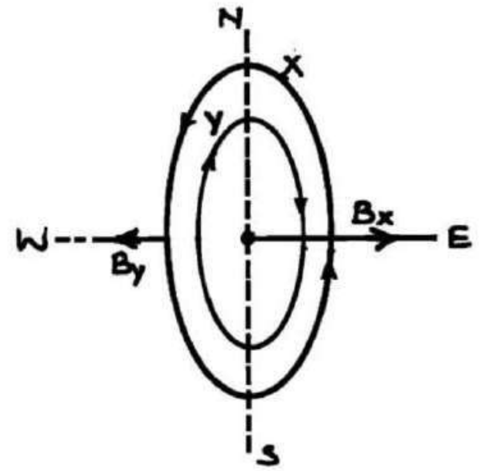
When point P lie at the centre of the circular coil ($x = 0$)

$$\boxed{B_0 = \frac{\mu_0 n I}{2a}}$$



Q. Two concentric circular coils X and Y of radius 16 cm and 10 cm respectively lie in the same verticle plane containing the north to south direction. Coil X has 20 turns and carries a current of 16 A. Coil Y has 25 turns and carries a current of 18 A. The sense of the current in X is anticlockwise and clockwise in Y, for an observer looking at the coils facing west. Give the magnitude and direction of the net magnetic field due to the coils at their centre.

Sol.



Radius of Coil X, $r_1 = 0.16\ m$

Radius of Coil Y, $r_2 = 0.1\ m$

Number of turns of coil X $n_1 = 20$

Number of turns of coil Y $n_2 = 25$

Current in coil X, $I_1 = 16\ A$

Current in coil Y, $I_2 = 18\ A$

Magnetic field due to coil X at their centre is

$$B_1 = \frac{\mu_0 n_1 I_1}{2r_1}$$

$$B_1 = \frac{4\pi \times 10^{-7} \times 20 \times 16}{2 \times 0.16} = 4\pi \times 10^{-4}\ T \text{ (Towards East)}$$

Magnetic field due to coil Y at their centre is given by -

$$B_2 = \frac{\mu_0 n_2 I_2}{2r_2}$$

$$B_2 = \frac{4\pi \times 10^{-7} \times 25 \times 18}{2 \times 0.1} = 9\pi \times 10^{-4}\ T$$

(Towards West)

Hence the net field is -

$$B = B_2 - B_1$$

$$B = 9\pi \times 10^{-4} - 4\pi \times 10^{-4} = 5\pi \times 10^{-4}\ T$$

or $B = 1.57 \times 10^{-3}\ T$ (Towards West)

Q. (CBSE 2008) An electron moves around the nucleus in a hydrogen atom of radius 0.51 Å with a velocity of 2×10^5 m/s. Calculate -

(a) The equivalent current due to orbital motion of electron.

(b) The magnetic field produced at the centre of the nucleus.

(c) The magnetic moment associated with the electron.

Sol. Given that - $r = 0.51\ \text{Å} = 0.51 \times 10^{-10}$

$$V = 2 \times 10^5 \text{ m/s}$$

(a) Time period $T = \dfrac{2\pi r}{V}$

Electric current $I = \dfrac{e}{T} = \dfrac{e}{2\pi r/V} = \dfrac{eV}{2\pi r}$

$$I = \frac{1.6 \times 10^{-19} \times 2 \times 10^5}{2 \times 3.14 \times 0.51 \times 10^{-10}} = 9.99 \times 10^{-5} \text{ A}$$

(b)
$$B = \frac{\mu_0 I}{2r} = \frac{\mu_0}{2r}\left(\frac{eV}{2\pi r}\right)$$

$$B = \frac{4\pi \times 10^{-7}}{2 \times 0.51 \times 10^{-10}} \times 9.99 \times 10^{-5}$$

or $$B = 1.23 \text{ T}$$

(c) Magnetic Moment $M = IA$

$$M = 9.99 \times 10^{-5} \times (\pi r^2)$$

$$M = 9.99 \times 10^{-5} \times 3.14 \times (0.51 \times 10^{-10})^2$$

or $$M = 8.16 \times 10^{-25} \text{ A} \cdot \text{m}^2$$

Q. (CBSE 2011, 10) A straight wire of length L is bent into a semi-circular loop. Use Biot-Savart Law to deduce an expression for the magnetic field at its centre due to the current I passing through it.

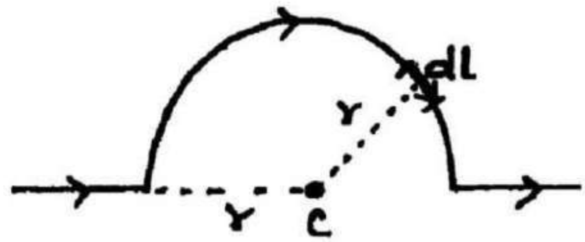
Sol. Length L is bent into semicircular loop.

Length of wire = Circumference of semicircular wire

$$L = \pi r$$

$$r = \frac{L}{\pi}$$

Considering a small element dl on current loop.



The magnetic field dB due to small current element Idl at centre C. Using Biot-Savart Law, we have -

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I dl \sin 90^\circ}{r^2} \qquad \left(\begin{matrix} I dl \perp r \\ \therefore\ \theta = 90^\circ \end{matrix} \right)$$

$$dB = \frac{\mu_0}{4\pi} \frac{I dl}{r^2}$$

Net magnetic field at C due to semicircular loop, -

$$B = \int_{\text{Semicircle}} \frac{\mu_0}{4\pi} \frac{I dl}{r^2}$$

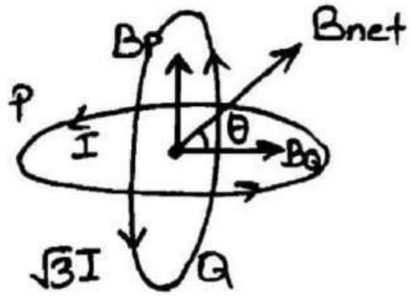
$$B = \frac{\mu_0}{4\pi} \frac{I}{r^2} \int_{\text{Semicircle}} dl = \frac{\mu_0}{4\pi} \cdot \frac{IL}{r^2}$$

But $r = \frac{L}{\pi}$ so that

$$B = \frac{\mu_0}{4\pi} \cdot \frac{IL}{(L/\pi)^2} \qquad \boxed{B = \frac{\mu_0 I \pi}{4L}}$$

Q. (CBSE 2016, 2019) Two identical coils P and Q each of radius R are lying in perpendicular planes such that they have a common centre. Find the magnitude and direction of magnetic field at the common centre of the two coils, if they carry currents I and $\sqrt{3}I$ respectively.

Sol. Magnetic field at the centre of coils P and Q are



$$B_P = \frac{\mu_0 I}{2R}$$

$$B_Q = \frac{\mu_0 \sqrt{3} I}{2R}$$

$$\therefore \quad B_{net} = \sqrt{B_P^2 + B_Q^2 + 2B_P B_Q \cos 90^\circ}$$

$$B_{net} = \sqrt{\left(\frac{\mu_0 I}{2R}\right)^2 + \left(\frac{\mu_0 \sqrt{3} I}{2R}\right)^2}$$

$$= \frac{\mu_0 I}{2R} \times 2 = \frac{\mu_0 I}{R}$$

$$\therefore \quad \tan\theta = \frac{B_P}{B_Q} = \frac{\frac{\mu_0 I}{2R}}{\frac{\mu_0 \sqrt{3} I}{2R}} = \frac{1}{\sqrt{3}}$$

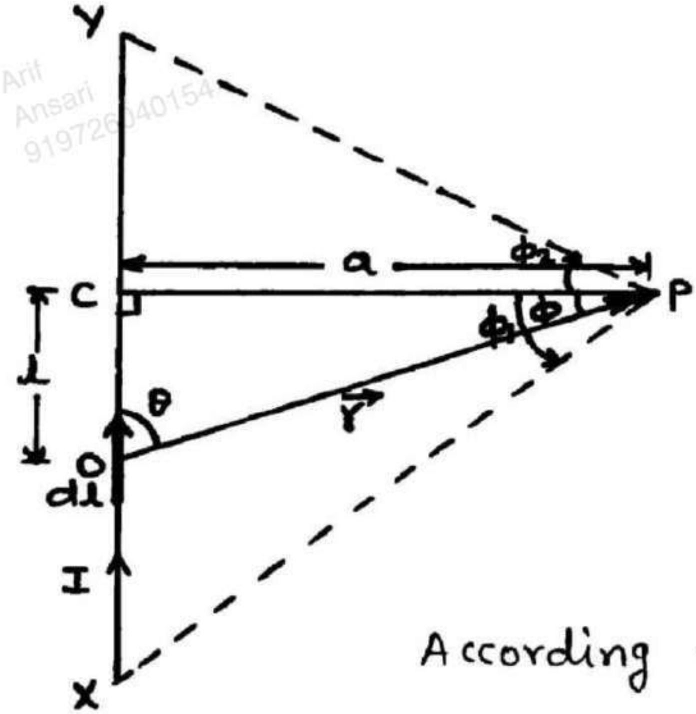
$$\theta = 30^\circ$$

[redacted]

Magnetic Field due to Straight wire carrying Current

Let P be a point at a perpendicular distance a from the wire. Let the conductor be made of small current elements.

Consider a small current element $I\vec{dl}$ of the wire at O. Here $\vec{r}$ is the position vector of P w.r.t. current element and θ be the angle between $I\vec{dl}$ and $\vec{r}$. Let CO = l.



$$\theta = 90^\circ - \phi$$

$$\sin\theta = \sin(90^\circ - \phi)$$

$$\sin\theta = \cos\phi$$

$$\cos\phi = \frac{a}{r} \quad \text{or} \quad r = \frac{a}{\cos\phi}$$

$$l = a\tan\phi$$

on differentiating it

$$dl = a\sec^2\phi\, d\phi$$

According to Biot-Savart's Law

$$dB = \frac{\mu_0}{4\pi} \times \frac{I\,dl\sin\theta}{r^2}$$

or $$dB = \frac{\mu_0}{4\pi} \frac{I\,(a\sec^2\phi\, d\phi)\cos\phi}{(a^2/\cos^2\phi)}$$

or $$dB = \frac{\mu_0}{4\pi}\frac{I}{a}\cos\phi\, d\phi$$

The total magnetic field at point P due to current through the whole straight conductor XY can be obtained by integrating dB within the limits $-\phi_1$ and $+\phi_2$.

$$B = \int_{-\phi_1}^{\phi_2} \frac{\mu_0 I}{4\pi a}\cos\phi\, d\phi$$

$$\boxed{B = \frac{\mu_0 I}{4\pi a}(\sin\phi_1 + \sin\phi_2)}$$

Special Cases -

(i) When the conductor XY is of infinite length and point P lies near the centre of the conductor ($\phi_1 = \phi_2 = 90^\circ$)

so $$B = \frac{\mu_0 I}{4\pi a}[\sin 90^\circ + \sin 90^\circ]$$

$$\boxed{B = \frac{\mu_0 I}{2\pi a}}$$

(ii) When the conductor XY is of infinite length but the point P lies near the end Y (or X) ($\phi_1 \cong 90^\circ$ and $\phi_2 \cong 0^\circ$)

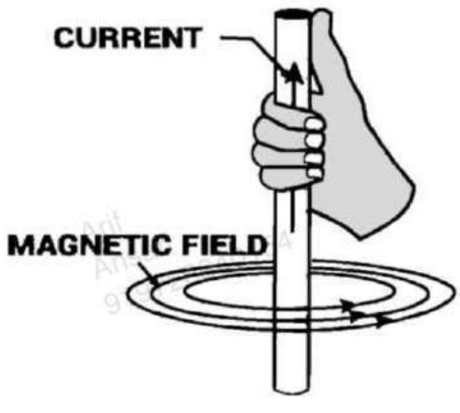
so $$B = \frac{\mu_0 I}{4\pi a}[\sin 90^\circ + \sin 0^\circ]$$

$$\boxed{B = \frac{\mu_0 I}{4\pi a}}$$

Direction of Magnetic Field

Right Hand Thumb Rule -

If we imagine the linear wire conductor to be held in the grip of the right hand so that the thumb points in the direction of current, then the curvature of the fingres around the conductor will represent the direction of magnetic field Lines.

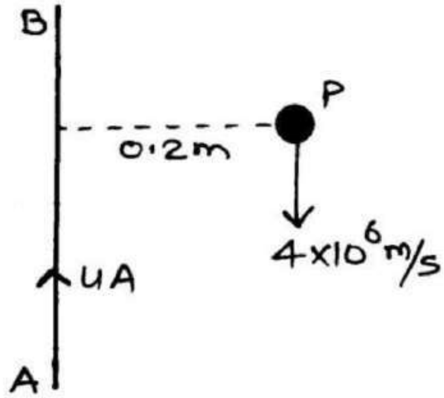


Q. A long straight wire AB carries a current of 4A. A proton travels at 4×10^6 m/s. parallel to the wire 0.2 m from it and in a direction opposite to the current. Calculate the force which the magnetic field due to the current carrying wire exerts on on the proton. Also specify its direction.

CBSE 2019

Sol. Given - $I = 4A$, $r = 0.2m$

$v = 4 \times 10^6$ m/s



Magnetic field at point P due to current carrying wire

$$B = \frac{\mu_0 I}{2\pi r}$$

Force on the moving proton

$$F = qvB\sin\theta$$

$$F = qv \times \frac{\mu_0 I}{2\pi r} \times \sin\theta$$

$$F = \frac{1.6 \times 10^{-19} \times 4 \times 10^6 \times 4\pi \times 10^{-7} \times 4 \times \sin 90^\circ}{2\pi \times 0.2}$$

$$F = 2.56 \times 10^{-18} N$$

Direction of force at point P is towards right. or away from the wire AB.

Ampere's Circuital Law

It states that the line integral of magnetic field $\vec{B}$ around a closed path in vacuum is equal to μ_0 times the total current I passing through the closed path.

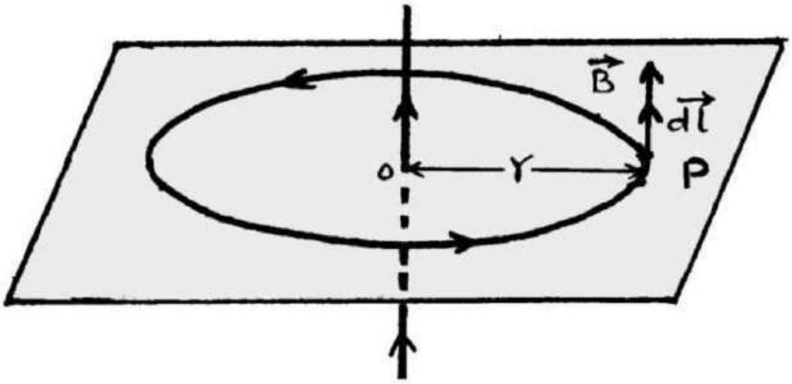
$$\boxed{\oint \vec{B}\cdot\vec{dl} = \mu_0 I}$$

Proof -

Consider a long thin straight wire XY carrying current I. The magnetic field lines around the Conductor are concentric circles in the plane perpendicular to the wire XY.

magnetic field produced at a point P at distance r $\quad B = \dfrac{\mu_0 I}{2\pi r}$

The direction of $\vec{B}$ is along the tangent to the magnetic field line at that point.



Consider a circular closed path of radius r whose centre O lie on the straight conductor.

Take a small element of length dL of the closed path at P. The direction of $\vec{B}$ and $\vec{dl}$ is the same.

Therefore line integral of $\vec{B}$ around the closed path of radius r is

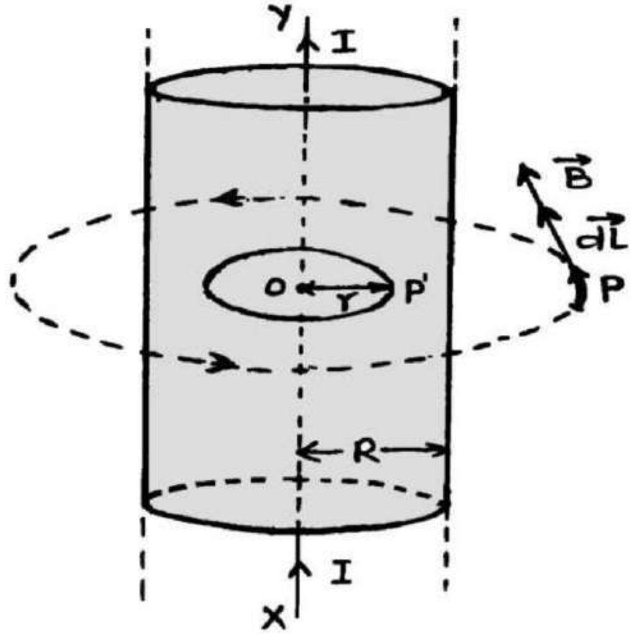
$$\oint \vec{B}\cdot\vec{dl} = \oint B\,dl\cos 0^\circ \Rightarrow \oint \vec{B}\cdot\vec{dl} = B\oint dL$$

$$= \frac{\mu_0 I}{2\pi r} \times 2\pi r \qquad \text{so} \qquad \boxed{\oint \vec{B}\cdot\vec{dl} = \mu_0 I}$$

Hence Proved.

Magnetic Field due to Current through a very Long Circular Cylinder -

Consider an infinite Long cylinder of radius R with axis XY. Let current I be the current passing through the cylinder. The magnetic field lines of force are perpendicular to the length of cylinder.



Case I - (Point P is lying outside the cylinder)

Let r be the perpendicular distance of point P from the axis of the cylinder (where $r > R$).

The magnetic field $\vec{B}$ is acting tangential to the magnetic line of force at P.

Here $\vec{B}$ and $\vec{dl}$ are acting in the same direction.

Applying Ampere circuital Law we have -

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

$$B\, 2\pi r = \mu_0 I \qquad \oint dl = 2\pi r$$

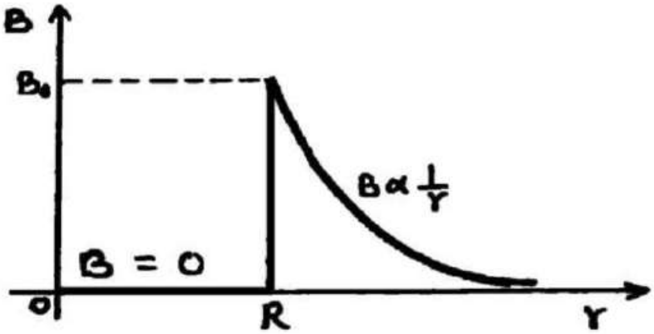
$$\boxed{B = \frac{\mu_0 I}{2\pi r}} \quad \text{i.e.} \quad \boxed{B \propto \frac{1}{r}}$$

Case II - (Point P is lying inside cylinder). $(r < R)$

We may have two possibilities-

(i) If the current is only along the surface of cylinder which is so if the conductor is a cylinderical sheet of metal, then current through the closed path L is zero.

Using Ampere circuital Law $\boxed{B = 0}$



(ii) If the current is uniformly distributed throughout the cross-section of the conductor, then the current through closed path is given by-

$$I' = \frac{I}{\pi R^2} \times \pi r^2 \quad = \frac{I r^2}{R^2}$$

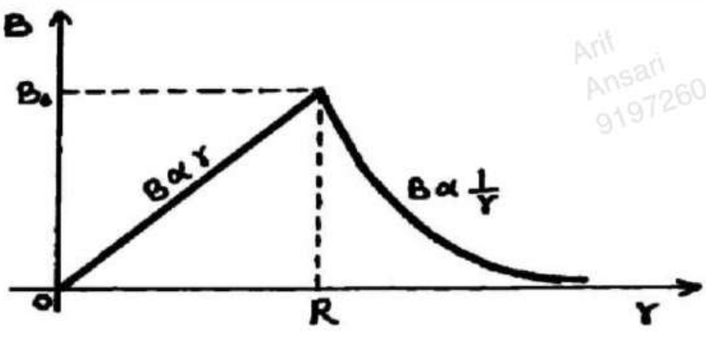
Applying Ampere's circuital Law, we have

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I'$$

$$B(2\pi r) = \mu_0 \frac{I r^2}{R^2}$$

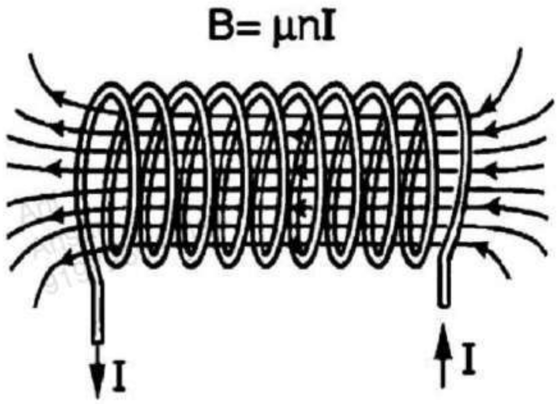
$$\boxed{B = \frac{\mu_0 I r}{2\pi R^2}} \qquad \boxed{B \propto r} \quad \text{for } r < R$$

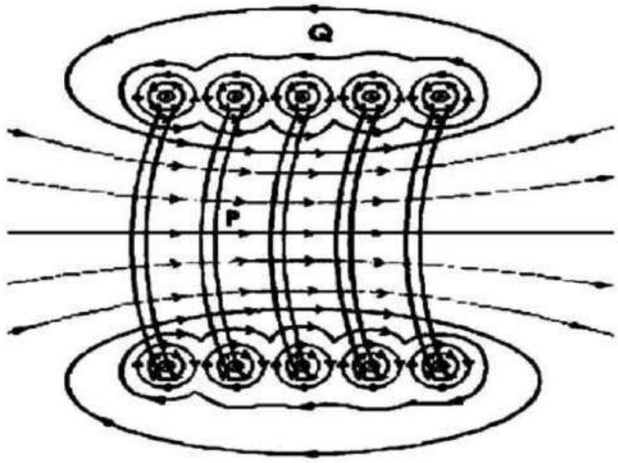
* So the magnetic field is maximum for a point on the surface of solid cylinder carrying current and is zero for a point on the axis of cylinder.



SOLENOID

A solenoid consist of an insulating Long wire closely wound in the form of a helix. Its Length is very large as compared to its diameter.





Magnetic Field due to a Solenoid-

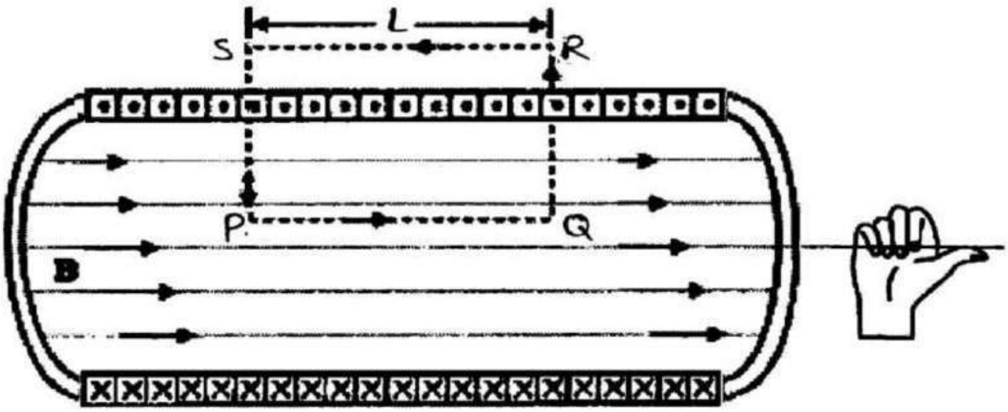
Consider a long straight solenoid of circular cross section. Each two turns of the solenoid are insulated from each other. When current is passed through the solenoid, then each turn of the solenoid can be considered as a circular Loop carrying current and thus will be producing a magnetic field.

At a point outside the solenoid, the magnetic fields due to neighbouring loops oppose each other and at a point inside the solenoid, the magnetic fields are in the same direction.

As a result of it the effective magnetic field outside the solenoid becomes zero, whereas the magnetic field inside the solenoid becomes strong and uniform and acting along the axis of solenoid.

Let n be the number of turns per unit length of solenoid.

Now consider a rectangular amperian loop PQRS near the middle of solenoid, where $PQ = l$.



Total numbers of turns in length l are $= nl$

The line integral of magnetic field induction $\vec{B}$ over the closed path PQRS is

$$\oint_{PQRS} \vec{B}\cdot\vec{dl} = \oint_P^Q \vec{B}\cdot\vec{dl} + \oint_Q^R \vec{B}\cdot\vec{dl} + \oint_R^S \vec{B}\cdot\vec{dl} + \oint_S^P \vec{B}\cdot\vec{dl}$$

$$\oint \vec{B}\cdot\vec{dl} = \int_P^Q Bdl\cos 0^\circ + \int_Q^R Bdl\cos 90^\circ + \int_R^S 0\cdot dl + \int Bdl\cos 90^\circ$$

$$\oint \vec{B}\cdot\vec{dl} = BL + 0 + 0 + 0$$

$$\oint_{PQRS} \vec{B}\cdot\vec{dl} = Bl$$

From Ampere's Circuital Law-

$$\oint_{PQRS} \vec{B}\cdot\vec{dl} = \mu_0 \times \text{total current through the rectangle PQRS}$$

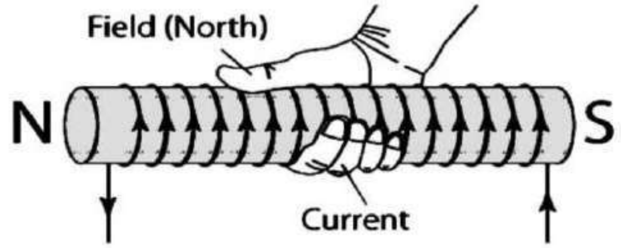
$$= \mu_0 \times \text{no. of turns in rectangle} \times \text{Current}$$

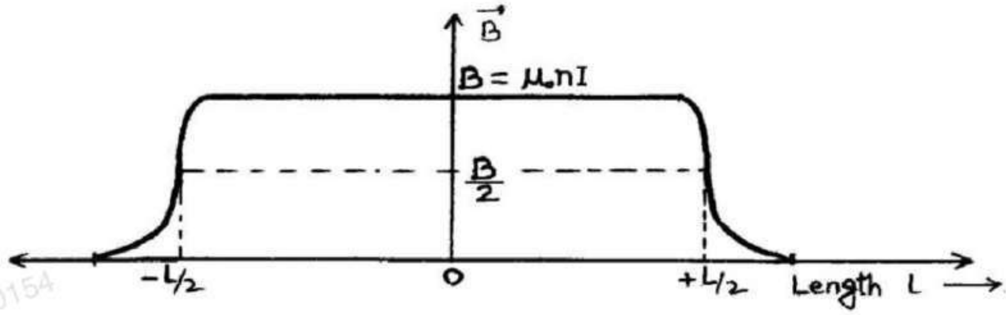
$$Bl = \mu_0 n l I$$

$$\boxed{B = \mu_0 n I} \qquad \boxed{B = \frac{\mu_0 N I}{L}}$$

It is the magnetic field inside the solenoid.

- ★ At a point near the end of a solenoid, the magnetic field is found to be $\frac{\mu_0 n I}{2}$
- ★ The Linear solenoid carrying current is equivalent to a bar magnet.





Q. A solenoid of length 1 m has a radius of 1 cm. and has a total 1000 turns wound on it. It carries a current of 5 A. Calculate the magnitude of the axial magnetic field inside the solenoid. If an electron was to move with a speed of 10^4 m/s along the axis of this current carrying solenoid, what would be the force experienced by this electron ?

Delhi 2008

Sol. Given - $L = 1\,m$, $r = 1\,cm = 10^{-2}\,m$.

$N = 1000$, $I = 5\,A$

$\therefore$ Magnetic field P inside the solenoid -

$$B = \mu_0 n I = \mu_0 \frac{N}{L} I$$

$$B = 4\pi \times 10^{-7} \times \frac{1000}{1} \times 5$$

$$B = 2\pi \times 10^{-3}\ T$$

Now $q = -e$, $v = 10^4$ m/s

and the angle between B and v is $0°$ because the electron moves along the direction of the magnetic field.

$\therefore$ Magnetic force $F_B = qvB \sin 0°$

So $F_B = 0$

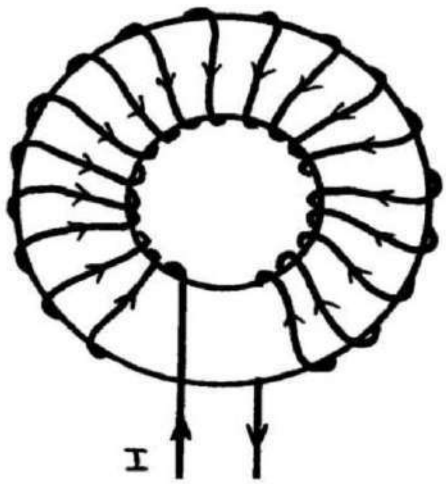
Hence No magnetic force experienced by the electron.

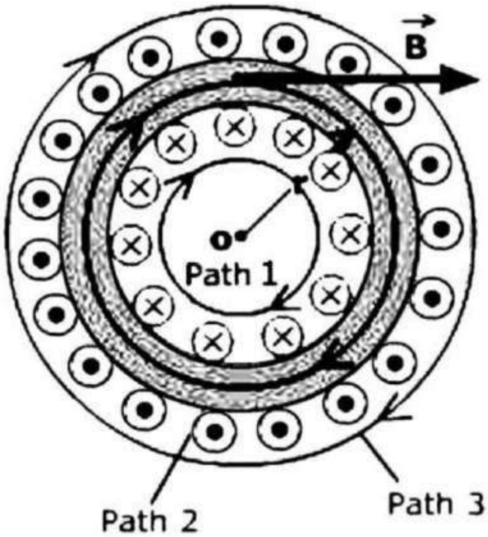
TOROID

The toroid is a hollow circular ring on which a large number of insulated wire turns are closely wound. Infact a toroid is an endless solenoid in the form of a ring.

Magnetic Field due to current in ideal toroid.

Let n be the number of turns per unit length of toroid and I be the current flowing through it. In case of ideal toroid, the coil turns are circular and closely wound.





The direction of the magnetic field at a point is given by the tangent to the magnetic field line at that point.

We draw three circular amperian loop 1, 2 and 3 of radius r_1, r_2 and r_3 to be traversed in clockwise direction,

So the magnetic field along loop 1 is.

$$\oint_{loop\,1} \vec{B_1}\cdot\vec{dl} = \mu_0 I$$

Here $I = 0$ for loop1 So $\boxed{B_1 = 0}$

The current enclosed by loop 3 is zero so the magnetic field along the loop 3 -

$$\oint_{loop\,3} \vec{B_3}\cdot\vec{dl} = \mu_0 \times 0$$

$$\boxed{B_3 = 0}$$

Let B is the magnetic field along the loop 2.

Current enclosed By the loop 2 =

= number of turns x Current in each turn

$= 2\pi r_2 n \times I$

So

$$\oint_{loop\,2} \vec{B}\cdot\vec{dl} = \mu_0 \times 2\pi r_2 n I$$

$$B \times 2\pi r_2 = \mu_0 \times 2\pi r_2 n I \qquad \left[\oint_{loop2} dl = 2\pi r_2\right]$$

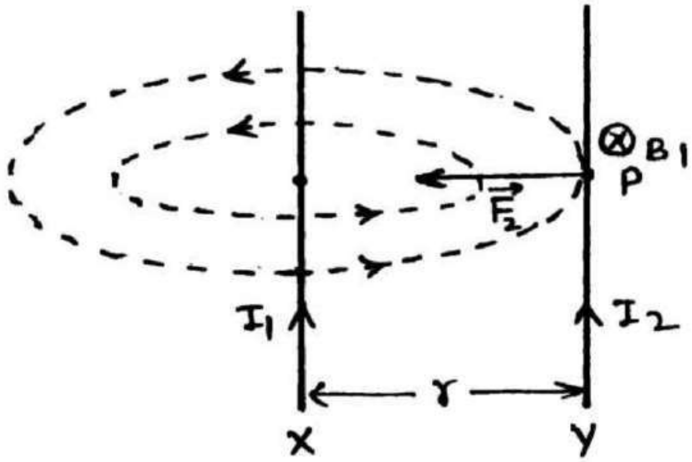
$$\boxed{B = \mu_0 n I}$$

This result is similar to that of solenoid.

Force between two parallel Current Carrying Conductors

Consider two infinite long straight conductor's carrying currents I_1 and I_2 in the same direction. They are held parallel to each other at a distance r.

Since magnetic field is produced due to current through each conductor, therefore each conductor experiences a force.



Magnetic field at a point P on conductor Y due to current I_1 passing through X is given by-

$$B_1 = \frac{\mu_0}{4\pi} \frac{2I_1}{r}$$

As the current carrying conductor Y lies in the magnetic field $\vec{B_1}$, therefore the unit length of Y will experience a force given by-

$$F_2 = B_1 I_2 \sin\theta \qquad [L = 1 \text{ (unit length)}]$$

$$F_2 = B_1 I_2 \times 1 = B_1 I_2$$

Putting the value of B_1, we have-

$$\boxed{F_2 = \frac{\mu_0}{2\pi} \times \frac{I_1 I_2}{r}}$$

According to Fleming's Left Hand Rule, force on conductor Y acts perpendicular to Y, directed towards X.

Similarly the conductor X also experiences a force F_1.

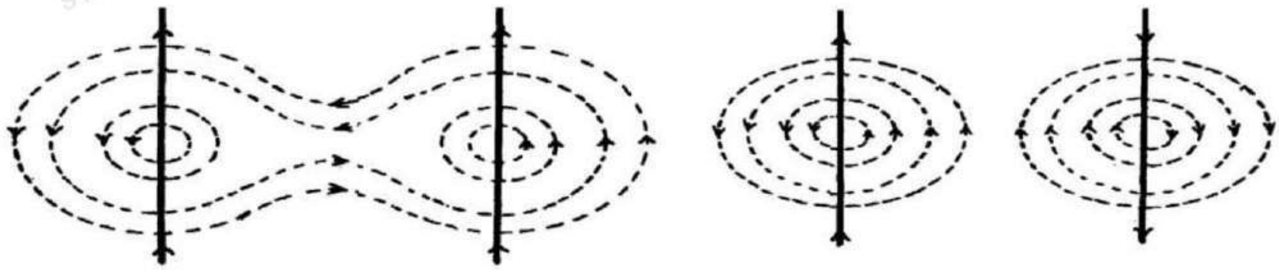
$$F_1 = B_2 I_1 \sin\theta \qquad [l = 1 \text{ (unit length)}]$$

$$= B_2 I_1 \times 1 = B_2 I_1$$

$$\boxed{F_1 = \frac{\mu_0}{2\pi} \times \frac{I_1 I_2}{r}} \qquad \left(B_2 = \frac{\mu_0}{2\pi}\frac{I_2}{r}\right)$$

F_1 acts perpendicular to x and directed towards y. Hence x and y attract each other.

★ Two parallel conductors carrying currents in the same direction attract each other.



★ If the currents in conductor x and y are in opposite directions then they repel each other.

Standard defination of 1 Ampere

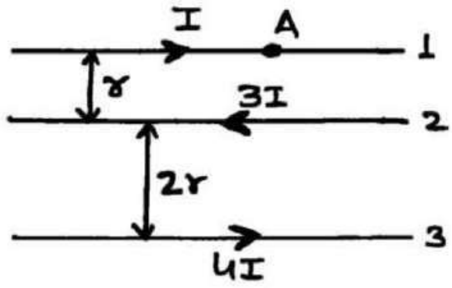
One ampere is that current which when flows through two parallel wires separated by 1 meter in vaccum produces a force of 2×10^{-7} N on unit length of wire.

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

If $l = 1$ m, $I_1 = I_2 = 1$ A and $r = 1$ m then –

$$F = 2 \times 10^{-7} \text{ N}$$

Q. CBSE 2017 The fig. shows three identical wires carrying current. Find - (i) Magnitude and direction of the net magnetic field at point A on conductor 1.
(ii) Magnetic force on conductor 2.



Sol. (i) $B_2 = \frac{\mu_0 (3I)}{2\pi r}$

into the plane of paper

$$B_3 = \frac{\mu_0 (4I)}{2\pi (3r)} = \frac{\mu_0 (2I)}{3\pi r}$$

out of the plane of paper

$$B_A = B_2 - B_3$$

$$B_A = \frac{\mu_0 (3I)}{2\pi r} - \frac{\mu_0 (2I)}{3\pi r}$$

$$B_A = \frac{\mu_0 [5I]}{6\pi r}$$ into the paper

(ii) Magnetic force per unit length on wire 2

$$F = \frac{\mu_0 \times (3I) \times I}{2\pi r} - \frac{\mu_0 \times (4I) \times (3I)}{2\pi (2r)}$$

$$= \frac{3}{2}\frac{\mu_0 I^2}{\pi r} - \frac{3\mu_0 I^2}{\pi r}$$

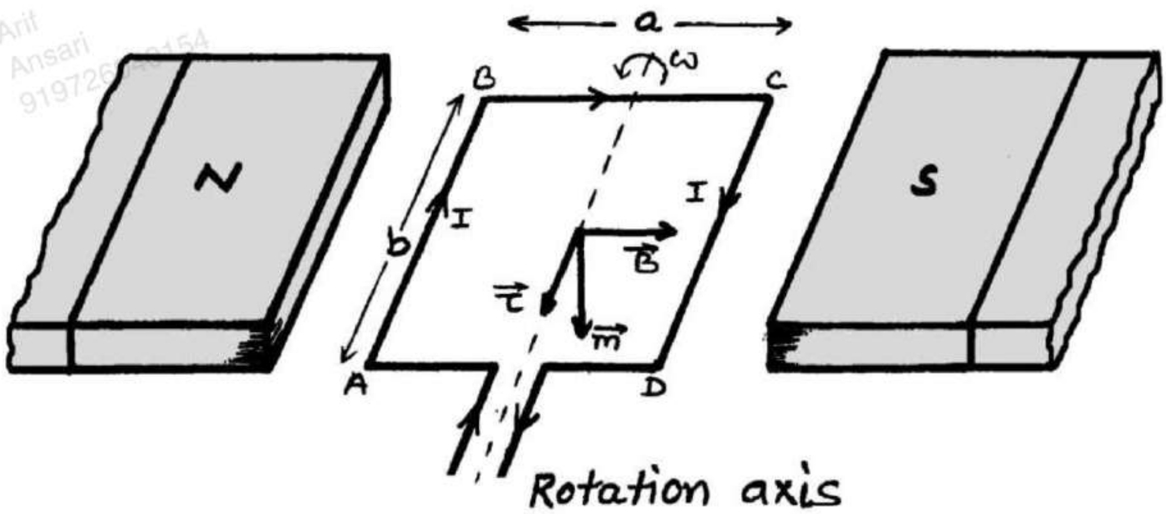
$$= -\frac{3}{2}\frac{\mu_0 I^2}{\pi r}$$

Hence Force F is in the direction of wire 1.

Torque on a current carrying Coil in a Magnetic Field -

When a rectangular current carrying coil is placed in a uniform magnetic field then it experiences a torque. It does not experience a net Force.

We first consider the simple case when the rectangular loop is placed such that the uniform magnetic field B is in the plane of the loop.

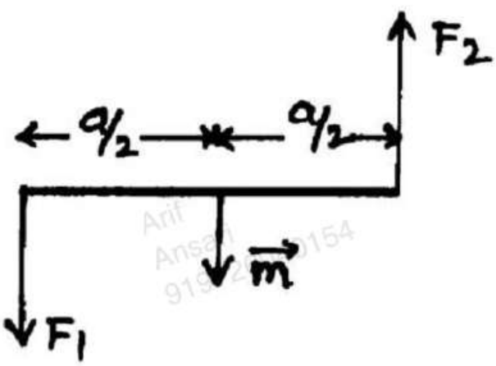


The field exerts no force on the two arms AD and BC of the loop.

The magnetic field is perpendicular to the arm AB of the loop and exerts a force F_1 on it, which is directed into the plane of the loop.

$$\boxed{F_1 = IbB}$$

similarly the magnetic field exerts a force F_2 on the arm CD, which is directed out of the plane of the loop.

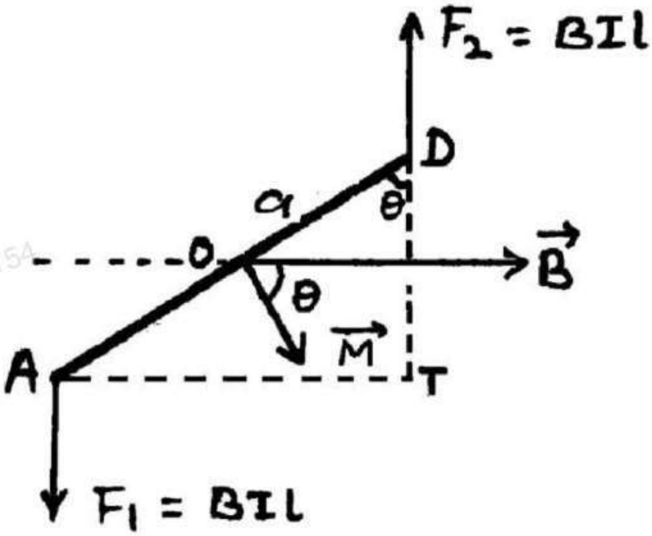


$$\boxed{F_2 = F_1 = IbB}$$

Thus the net force on the loop is zero.

Now consider the case when the plane of the loop, is not along the magnetic field and makes an angle with it.

Let the angle between the field and the normal to the coil be angle θ.



$\sin\theta = \frac{AT}{a}$

$AT = a\sin\theta$

The force on arms AB and CD are F_1 and F_2.

$$F_1 = F_2 = IBb.$$

So the magnitude of the torque on the loop is

$$\tau = F \times a\sin\theta = IBba\sin\theta$$

$$\tau = IAB\sin\theta$$

$$\vec{\tau} = I(\vec{A} \times \vec{B})$$

If the loop has N closely wound turns, then the torque will be -

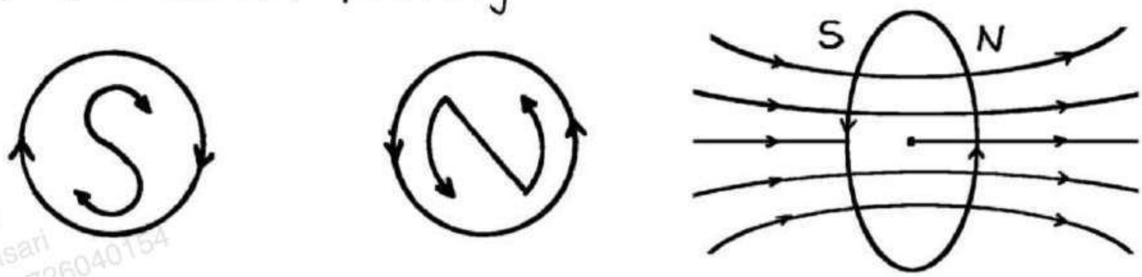
and
$$\boxed{\begin{aligned} \vec{\tau} &= NI(\vec{A} \times \vec{B}) \\ \vec{M} &= NI\vec{A} \end{aligned}}$$

$\vec{M}$ = magnetic moment of the current loop.

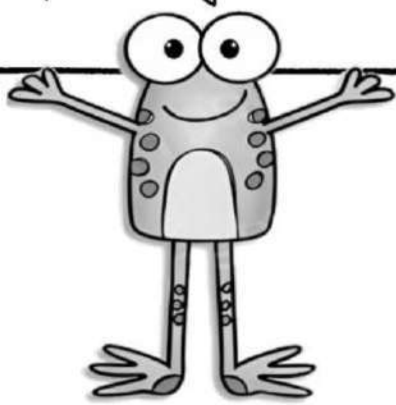
SI Unit of magnetic moment is $A\text{-}m^2$.

Polarity of the magnetic dipole moment -

Look at one face of the coil. If the direction of current through the coil is clockwise then the face has south polarity.



If the direction of current is anticlockwise, then that face has north polarity.



Arif Ansari 919726040154

Q. A square coil of side 10 cm. has 20 turns and carries a current of 12 A. The coil is suspended vertically and the normal to the plane of the coil, makes an angle θ with the direction of a uniform horizontal magnetic field of 0.8 T. If the torque, experienced by the coil equals 0.96 N-m, find the value of θ.

Delhi 2010

Sol. Here area of the coil $= 10 \times 10 = 100\ cm^2 = 10^{-2}\ m^2$

Number of turns $N = 20$, Current $= 12$ A

Magnetic field $B = 0.8$ T

Torque $= 0.96$ N-m.

Torque experienced by current carrying coil in the magnetic field is

$$\tau = NIAB \sin\theta$$

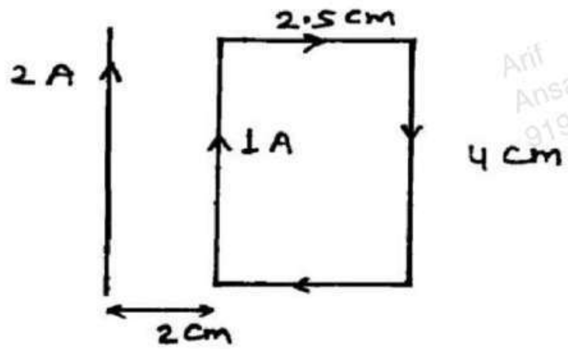
$$0.96 = 20 \times 12 \times 10^{-2} \times 0.8 \times \sin\theta$$

$$\sin\theta = \frac{0.96}{1.92} = \frac{1}{2}$$

so $$\theta = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6} \text{ rad.}$$

Q. A rectangular loop of wire of size 2.5 cm x 4 cm. carries steady current of 1 A. A straight wire carrying 2 A current is kept near the loop as shown. If the loop and the wire are coplanar, find the (i) torque acting on the loop and (ii) the magnitude and direction of force on the loop due to the current carrying wire.

CBSE 2012



Sol. There will be force of attraction between the straight wire and the 4 cm long arm of loop nearer to straight conductor.

$$F_1 = \frac{\mu_0}{4\pi} \frac{2 \times 2 \times 1}{(2 \times 10^{-2})} \times 4 \times 10^{-2}$$

$$F_1 = 8 \times 10^{-7} \text{ N} \quad \text{(towards straight wire)}$$

Similarly the force on other 4 cm arm of loop away from the straight conductor

$$F_2 = \frac{\mu_0}{4\pi} \frac{2 \times 2 \times 1}{(4.5 \times 10^{-2})} \times 4 \times 10^{-2}$$

$$F_2 = 3.55 \times 10^{-7} \text{ N} \quad \text{(away from straight wire)}$$

(i) F_1 and F_2 are of different magnitude therefore do not form couple and hence

$$\text{Torque} \quad \tau = 0$$

(ii) Net force on loop

$$F = F_1 - F_2 \quad \text{(towards straright wire)}$$

$$F = 8 \times 10^{-7} - 3.55 \times 10^{-7}$$

$$F = 4.45 \times 10^{-7} \text{ N}$$

The force on two branches of loop are equal in magnitude, opposite in direction, hence they balance each other.

Moving Coil GALVANOMETER

Moving coil galvanometer is an instrument used for detection and measurement of small electric current and voltage.

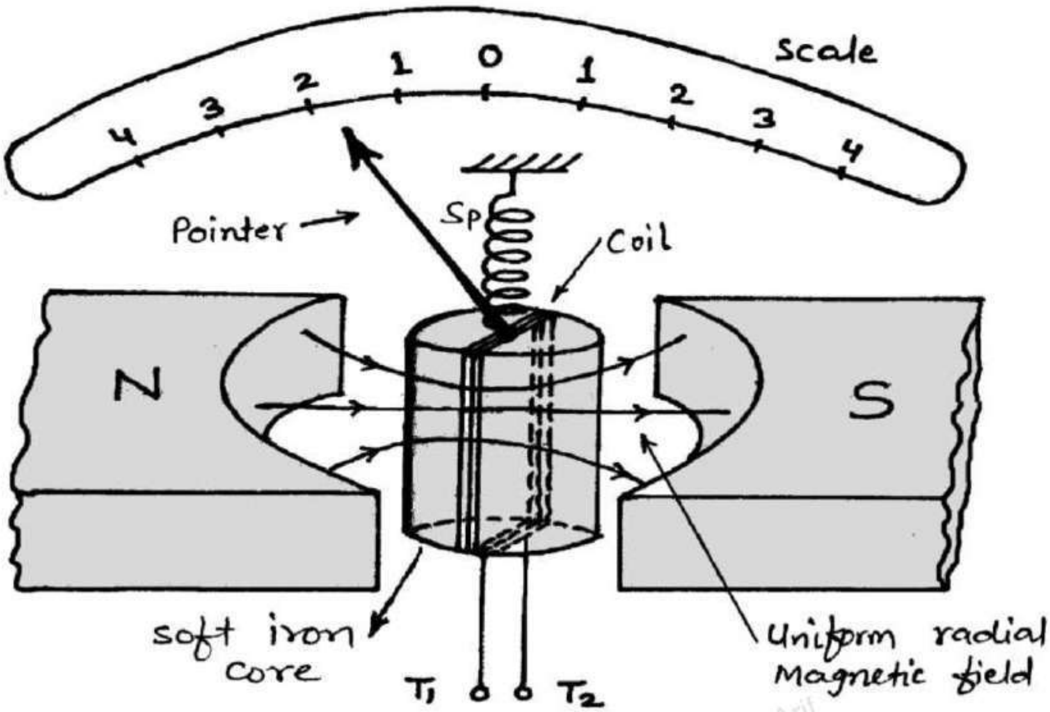
Principle -

Its working is based on the fact that when a carrying coil is placed in a magnetic field, it experiences a torque.

Construction -

The galvanometer consist of a coil with many turns, which is free to rotate about a fixed axis in a uniform radial magnetic field.

There is a cylinderical soft iron core, which increases the strength of the magnetic field and makes the field radial.



Working :-

When the current flows through the coil, a torque acts on it. Which can be given as -

$$\boxed{\tau = NIAB} \qquad (\sin\theta = 1)$$

Since the field is radial by dusign, so we have taken $\sin\theta = 1$.

This magnetic torque tends to rotate the coil.

The spring Sp provides a counter torque $K\phi$ that balance the magnetic torque NIAB and results in steady angular deflection ϕ.

In equilibrium -

$$K\phi = NIAB$$

Where K is the torsional constant of the spring i.e. restoring torque per unit twist.

The deflection ϕ is indicated on the scale by a pointer attached to the spring.

We have $$\boxed{\phi = \left(\frac{NBA}{K}\right) I}$$

Hence the deflection produced is proportional to the current flowing through the galvanometer.

Current Senitivity -

It is defined as the deflection produced in the galvanometer when a unit current flows through it.

$$\boxed{\text{Current Sensitivity } (I_s) = \frac{\phi}{I} = \frac{NBA}{K}}$$

The unit of current sensitivity is rad./A or division/A.

Voltage Sensitivity -

It is defined as the deflection produced in the galvanometer when a unit voltage is applied across the two terminals of the galvanometer.

$$\boxed{\text{Voltage Sensitivity } (V_s) = \frac{\phi}{IR} = \frac{NBA}{KR}}$$

The unit of voltage sensitivity is rad/V or division/V.

Q. (All India 2008) Why should the spring/suspension wire in a moving coil galvanometer have low torsional constant?

Sol. Low torsional constant facilitate greater deflection (α) in coil for given value of current and hence sensitivity of galvanometer increases.

Q. (CBSE 2006) Write two factors by which current sensitivity of a galvanometer can be increased.

Sol. Current sensitivity of galvanometer can be increased by -

(i) Increasing the number of turns of coil
(ii) Increasing the magnetic field.

Q. (CBSE 2008) Write two factors by which Voltage sensitivity of a moving coil galvanometer can be increased.

Sol. Voltage sensitivity of galvanomer can be increased by

(i) Increasing the magnetic field
(ii) Decreasing the value of torsional Constant.

Q. (RBSE 2005) The coils in certain galvanometer, have a fixed Core made of a non-magnetic metallic material. Why does the oscillating coil come to rest so quickly in such a Core?

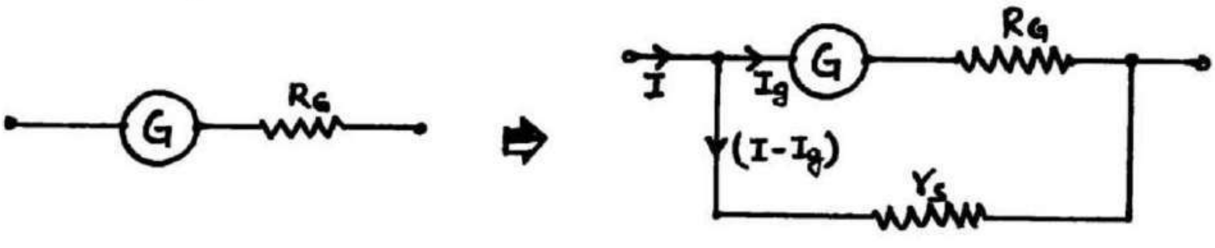
Sol. Due to eddy currents produced in core which opposes the cause (deflection of coil), that produces it.

Ammeter

- An ammeter is a low resistance galvanometer.
- It is used to measure the current in a circuit in amperes.

Galvanometer is a very sensitive device, it gives a full scale deflection for a current of the order of μA. Also for measuring currents, the galvanometer has to be connected in series and as it has a large resistance, this will change the value of the current in the circuit.

To overcome these difficulties a low resistance wire r_s, called shunt resistance is connected in parallel with the galvanometer coil, so that most of the current passes through the shunt.



Ammeter

Effective resistance of converted galvanometer into ameter -

$$\boxed{R = \frac{R_G\, r_s}{R_G + r_s}}$$

Also Here - $I_g R_G = (I - I_g) r_s$

so

$$\boxed{r_s = \frac{I_g}{(I - I_g)} R_G}$$

★ An ideal ammeter has zero resistance.

Voltmeter

A voltmeter is a high resistance galvanometer.

It is used to measure the potential difference between two points of a circuit in volts.

A galvanometer can be converted into a voltmeter by connecting a high resistance in series with the galvanometer. So that it draws a small current, otherwise the voltage measurement will disturb the original set up by an amount which is very large.

Effective resistance of converted galvanometer into Voltmeter -

$$\boxed{R_s = R_G + R}$$

Also from ohm's Law - $I_g = \dfrac{V}{R_G + R}$

$$\boxed{R = \frac{V}{I_g} - R_G}$$

★ The resistance of an ideal voltmeter is infinity.

Q. All India 2007

A galvanometer has a resistance of 30 Ω. It gives full scale deflection with a current of 2 mA. Calculate the value of the resistance needed to convert it into an ammeter of range 0 - 0.3 A

Sol: Resistance of galvanometer $R_G = 30\ \Omega$

Range of galvanometer $I_g = 2\text{mA} = 2 \times 10^{-3}$ A

Range of ammeter $I = 0.3$ A

A galvanometer of range I_g and resistance R_G can be converted into an ammeter by connecting a very low resistance r_s in parallel with the galvanometer where value is given by -

$$r_s = \frac{I_g R_G}{I - I_g}$$

$$r_s = \frac{2 \times 10^{-3} \times 30}{(0.3 - 2 \times 10^{-3})} = \frac{6 \times 10^{-2}}{0.298} = 0.2\ \Omega$$

Q. CBSE 2006

Which one of the two an ammeter or a milliammeter, has higher resistance and why?

Sol. The milliammeter has higher resistance.

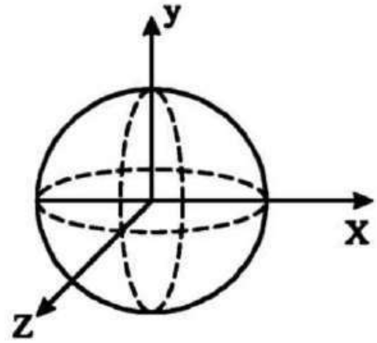
∵ Higher the range of ammeter, smaller the value of shunt resistance. The equivalent resistance of parallel combination of the galvanometer and shunt resistance is smaller than the shunt resistance.

Ammeter has got higher range, therefore smaller value of shunt resistance and smaller equivalent resistance.

Q.1. Three rings, each having equal radius R, are placed mutually perpendicular to each other and each having its centre at the origin of coordinate system. If current I is flowing through each ring then find the magnitude of the magnetic field at the common centre.

Sol:

B due to the ring lying in XY-plane is



$$B_{xy} = \frac{\mu_0 I}{2R} \text{ along Z-axis.}$$

B due to the ring lying in YZ-plane is $B_{yz} = \frac{\mu_0 I}{2R}$ along X-axis and B due to the ring lying in XZ-plane is $B_{xz} = \frac{\mu_0 I}{2R}$ along Y-axis.

$$\therefore \vec{B}_{net} = \frac{\mu_0 I}{2R}\left(\hat{i}+\hat{j}+\hat{k}\right) \Rightarrow B_{net} = \sqrt{3}\frac{\mu_0 I}{2R}$$

Q.2. Two circular coils are made from a uniform wire. The ratio of radii of circular coils are 2:3 and ratio of number of turns is 3:4. If they are connected in parallel across a battery.

(a) Find ratio of magnetic inductions at their centres

(b) Find the ratio magnetic moments of 2 coils.

Sol: When connected in parallel

$$a) B \alpha \frac{1}{r^2}; \frac{B_1}{B_2} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$b) M = ni\, A_{coil} \text{ but } i = \frac{V}{R} = \frac{V}{\rho l} A_{wire}$$

$$\Rightarrow M = \frac{V}{\rho(2\pi r_{coil})}(\pi r_{coil}^2);\ M = \frac{V}{\rho X 2} r_{coil}$$

$$\frac{M_1}{M_2} = \frac{r_1}{r_2} = \frac{2}{3}.$$

Q.3. A magnetic field of $\left(4.0\times10^{-3}\hat{k}\right)T$ exerts a force $\left(4.0\hat{i}+3.0\hat{j}\right)\times10^{-10}N$ on a particle having a charge $10^{-9}C$ and moving in the x-y plane. Find the velocity of the particle.

Sol: Magnetic force $\overrightarrow{F_m} = \left(4.0\hat{i}+3.0\hat{j}\right)\times10^{-10}N$

Let velocity of the particle in x-y plane be

$\vec{v} = v_x\hat{i} + v_y\hat{j}$.

Then from the relation $\overrightarrow{F_m} = q\left(\vec{v}\times\vec{B}\right)$

$$\left(4.0\hat{i}+3.0\hat{j}\right)\times10^{-10} = 10^{-9}\left[\left(v_x\hat{i}+v_y\hat{j}\right)\times\left(4\times10^{-3}\hat{k}\right)\right]$$

comparing the coefficient of $\hat{i}$ *and* $\hat{j}$ we have,

$$4\times10^{-10} = 4v_y\times10^{-12}$$

$$\therefore v_y = 10^2 m/s = 100m/s \text{ and}$$

$$3.0\times10^{-10} = -4v_x\times10^{-12}$$

$$\therefore v_x = -75m/s \quad \therefore \vec{v} = -75\hat{i}+100\hat{j}$$

Q.4. A long straight conductor carrying a current of 2A is in parallel to another conductor of length 5cm and carrying a current 3A. They are separated by a distance of 10cm. Calculate (a) B due to first conductor at the location of second conductor (b) the force on the short conductor.

Sol. Given $i_1 = 2A; i_2 = 3A$

$r = 10cm = 10\times10^{-2}m; l_2 = 5cm$

a) $B = \frac{\mu_0 i_1}{2\pi r} = 2\times10^{-7}\times\frac{2}{10\times10^{-2}} = 4\times10^{-6}$ Tesla

b) $F = \frac{\mu_0 i_1 i_2}{2\pi r} \times \ell_2$

$$= 2\times10^{-7} \times \frac{2\times3}{10\times10^{-2}} \times 5\times10^{-2} \quad = 6\times10^{-7}\,N$$

Q.5. A galvanometer of resistance 20Ω is shunted by a 2Ω resistor. What part of the main current flows through the galvanometer?

Sol. $\frac{i_g}{i} = \frac{S}{G+S}$. Given $G = 20\Omega; S = 2\Omega$

$\therefore \frac{i_g}{i} = \frac{2}{22} = \frac{1}{11}; \frac{1}{11}th$ part of current is passing through galvanometer.

Q.6. The resistance of galvanometer is 999Ω. A shunt of 1Ω is connected to it. If the main current is $10^{-2}\,A$, what is the current flowing through the galvanometer.

Sol. $G = 999\Omega, S = 1\Omega\ i = 10^{-2}\,A; \quad i_g = ?$

$$i_g = i\left(\frac{S}{G+S}\right) = 10^{-2} \times \left(\frac{1}{999+1}\right) = 10^{-5}\,A$$

Q.7. A maximum current of 0.5mA can be passed through a galvanometer of resistance 20Ω, Calculate the resistance to be connected in series to convert it into a voltmeter of range $(0-5)V$.

Sol. $R = G(n-1), where\ n = \frac{V}{V_g}$

$$V = 5V; V_g = i_g G = 0.5\times10^{-3} \times 20 = 10^{-2}\,V$$

$$\therefore n = 500 \quad \text{and } R = 20(500-1) = 9980\Omega$$